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AUTHOR McLeod, John
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ABSTRACT

Rational, defensible guidelines for use in identifying learning disabled children would assist in determining the most efficient use of diagnostic expertise, in specifying learning disabled populations for research purposes, and in assuring equity in the provision of diagnostic and remedial services. One commonly accepted symptom of learning disability is a discrepancy between a child's scores on an intelligence measure and on measures of educational achievement. However, many formulae that purport to indicate such discrepancies have proven arbitrary. A psychometric approach to defining learning disability recognizes that if the correlation is known between an intelligence test and an educational achievement test, it is possible to predict an individual's achievement level from the measured level of intelligence. It is also possible to estimate the error of this prediction. The assessment of an individual's measured intelligence level is itself subject to an error that can be estimated, as can the error of assessed achievement level. By consolidating the compounded errors, the statistical significance of the difference between the individual's predicted and measured achievement can be estimated. If the difference between the two is such that it could not be the result of chance, then the individual should be regarded as potentially learning disabled. Appendixes contain data from a computer program used to generate cut-off levels of achievement for students of different IQ levels, illustrative tables, and a discussion of the derivation of the formula used to define the discrepancy between predicted and measured achievement. (Author/PL)

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Psychometric Identification of Children with
Learning Disabilities

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by
John McLeod, Ph.D.

Institute of Child Guidance and Development

University of Saskatchewan
January, 1973

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ABSTRACT

Diagnosis of learning disability is, or should be, an individualized process. However, there is a need for rational, defensible, guidelines for the purpose of initial identification or screening of children who are likely, as a result of further individual examination, to be diagnosed as learning disabled.

Some of the needs for such guidelines are:

- (1) efficient use of diagnostic expertise;
- (2) specification of learning disabilities populations which will permit replication of, and generalization from, research projects; and
- (3) equitability in provision of diagnostic and remedial services.

A common and accepted symptom of learning disability is a discrepancy between score on a test of measured intelligence and score on a test of measured educational achievement in a basic area such as reading or math.

There have been many formulae which purport to indicate significant "retardation relative to intelligence" but these have, for the most part, been arbitrary and ill-founded.

The present monograph attempts to present an approach to psychometric definition of learning disability which is methodologically sound. The approach recognizes that, if the correlation is known between a predictor test (usually an intelligence test) and a criterion test (usually an educational achievement test), it is possible to predict an individual's achievement level from his or her measured level of intelligence, and it is also possible to estimate the error of this individual prediction. The assessment of an individual's measured intelligence level is itself subject to an error which can be estimated, as can the error of assessed achievement level.

By consolidating the compounded errors, the statistical significance of an observed difference between an individual's predicted achievement and his or her measured achievement can be estimated. If the difference between an individual's predicted and observed achievement is such that it could be expected to arise so infrequently as not to be likely due to chance, then the individual should be regarded as potentially learning disabled, subject to individual diagnostic confirmation.

The actual formula defining the discrepancy, (D), between predicted and observed achievement is:

$$D = \sigma \sqrt{2 - r_e - r_p \cdot r^2}$$

where σ = standard deviation of scores on the criterion (achievement)
test

r_c = reliability of the criterion test

r_p = reliability of the predictor (IQ) test

r = correlation between predictor and criterion tests.

Illustrative tables are provided, together with a computer program which will generate cut-off levels of achievement for students of different IQ levels.

FOREWORD

I am glad to recommend this monograph by Dr. John McLeod to the attention of Directors of Special Education or Guidance, and to School Psychologists whose responsibility it is to assess the abilities and achievements of retarded students and to plan appropriate provision for them. So many erroneous and misleading definitions of "under-achievement" and/or "learning disabilities" are currently being used by educational psychologists and administrators that it is high time that the basic psychometric principles should be clearly presented.

It is unfortunate that, in the 1920s, many of the pioneers of intelligence testing made such exaggerated claims for the predictive value of Mental Ages and IQs. Intelligence was thought to be fixed by heredity, and thus to be the main determinant of children's scholastic success or failure. Thus the Accomplishment or Achievement Quotient, i.e.: $100 \times \frac{\text{Educational Quotient}}{\text{Intelligence Quotient}}$ was put forward as a measure of the extent to which the child was "working up to his potential capacity". However psychometrists soon pointed out that this was statistically absurd (for the reasons that Dr. McLeod explains), and that in practice there were as many "over-achievers" as "under-achievers", i.e. children who were apparently achieving better than they were innately capable of doing. Nowadays it is generally accepted that intelligence, as well as achievement, depends partly on environmental advantage or handicap, as well as on heredity; hence there is no reason why such children should not score lower on the intelligence than achievement. Achievement Quotients and the like were banished by competent mental testers 30 years or more ago; and yet they still persist among many School Psychologists, and are even enshrined in the regulations of some State and Federal Education Departments.

Nevertheless intelligence tests (especially individual tests given by trained testers) are still quite useful predictors of likely educational achievement over the next few years, though by no means the only factor. The only statistically sound technique of making such predictions involves use of the regression equation, as described in the present monograph. It is most desirable, therefore, that people employed in such work should read and understand Dr. McLeod's arguments. However, if they do get lost in some of his statistical equations, he has provided special tables which give the predicted achievement level of children of any IQ at any age from 6 to 14 years, together with the cut-off points for designating the most severely retarded cases who need special treatment or placement.

Philip E. Vernon

Professor of Educational Psychology,
University of Calgary, Alberta.
Emeritus Professor of Psychology,
University of London.

PREFACE

Educational underachievement is perhaps one of the most misunderstood and misused concepts in educational psychology. It is hoped that the present monograph, apart from providing useful and tangible guidelines for special educators, will also help correct some psychometric misconceptions. However, as the popular folklore surrounding "underachievement" has proved resistant to enlightenment from mathematical, educational and psychological giants in the past, the present author has no illusions that he is going to be able to confuse the unconfusable with facts.

The text, opinions, calculations and errors in this monograph are those of the author alone, but he is indebted to: (a) the giants referred to in the preceding paragraph, such as Maynard Reynolds, McVicker Hunt, Robert Thorndike and Francis Galton, for their leadership, scholarship and influence; and (b) to Leonard Haines and Bikkar Randhawa for constructive suggestions and criticisms of the initial draft of the monograph.

One person who may legitimately be classified under both categories (a) and (b) is Philip Vernon, presently of Calgary and formerly the illustrious Professor of Educational Psychology at the University of London, England. It was Professor Vernon who first opened my eyes, nearly a quarter of a century ago, to the simplistic - and incorrect - general educational psychological practices which were being used to identify what were then referred to as "specifically retarded readers", many of whom would now be labelled "learning disabled".

The procedures suggested in the present monograph involve some algebra - not particularly advanced algebra - but certainly more complicated than the "Achievement Age (ought to) = Measured Mental Age" which, incredibly, is still prevalent.

In his reaction to the initial draft of the present monograph, Vernon ruefully commented that "administrators or psychologists concerned with picking out serious learning disabilities may find it rather difficult to follow the statistics and therefore . . . revert to simpler equations like the Michigan one." My acquired Australian reaction is: "Too (expletive deleted) right, mate!"

But one lives in hope . . .

John McLeod

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Definitions of learning disability

Insofar as it is possible to describe learning disabilities meaningfully, it has been argued (Cruickshank, 1977) that it is essential that a definition be based on diagnosed or assumed malfunctions of the neurological system of the organism.

Cruickshank is unsympathetic to "quantitative applications and regulations of an administrative nature (that have often been) confused with definition". Nevertheless, particularly when administrators and legislators are being pressured to make tangible provision for the diagnosis and education of learning disabled children, it is not unreasonable that they themselves should demand some tangible identifying markers:

- (a) to provide an estimate of the number of children for whom provision has to be made;
- (b) to ensure that there is equality of opportunity for special educational programs -- and perceived equality -- rather than opportunity which might seem to depend on individual or corporate parental persuasiveness; and
- (c) to provide some sort of screening procedure which will help to ensure that services will be used efficiently.

Senf (1977) acknowledges another justification for a more psychometrically based definition:

There is yet another impediment to the orderly accumulation of knowledge involving both research and practice: the definition of those termed learning disabled. If we are to amass empirical data, despite its admitted limitations, we must know who is being studied. If researchers do not communicate to others the specific populations they study, we shall forever be in the awkward position of knowing highly detailed information about groups of children whose generality

is unknown. Thus, on the one hand, research requires a definitive specification of its population or else suffers the consequences of accumulating ungeneralizable information.

Unfortunately, there has recently been a plethora of formula "defining" learning disability, partly as a result of U.S. Public Law 94-142. Most of these formulae, including that of the Federal Register (1976), have been arbitrary at best and, at worst, have been indicative of an ignorance of basic psychometrics, or of the apparent indestructibility of some psychometric myths.

Some psychometric myths

Myth #1: Educational achievement age (or quotient) ought to equal measured mental age (or IQ). A recent publication of the Michigan State Education Department (1976) states:

Grade level expected achievement is established by means of a formula:

$$\frac{CA \times IQ}{100} - 5 = \text{Expected achievement grade}$$

...this method of determining expected achievement has proved through both practice and research, to be the best method of determining achievement in reading for the vast majority of students to whom it has been applied.

and so,

Myth #2: A difference between measured mental age and measured educational achievement age, or a difference between measured IQ and measured educational quotient (EQ) which exceeds some arbitrarily prearranged figure, is a valid index of "underachievement".

Some psychometric facts

Fact #1: Measured IQ (or Mental Age) is not a valid index of expected EQ (or Educational Age) except when a student's IQ is 100.

In 1888, Galton introduced the notion of co-relation (sic) which could be expressed quantitatively. The idea was developed by his student, Karl Pearson, whose "product-moment correlation co-efficient" is denoted by the letter "r", to signify "regression".

The phenomenon of regression, as Galton (1888) pointed out, and which has remained a mathematical inevitability for all to see, means that if a dependent variable (such as Educational Quotient) is predicted from a correlated measure (such as IQ) the predicted value of the dependent variable will, on average, regress toward the mean. This phenomenon is illustrated in the idealized distribution of scores set out in Figure 1.

Figure 1.
Idealised distribution of IQs and EQs

		IQ						frequency
		75	85	95	105	115	125	
EQ	125					2	2	4
	115				4	10	2	16
	105			4	16	4		24
	95		2	10	4			16
	85		2	2				4
	75							
freq.		4	16	24	16	4		64

In order to make meaningful comparisons between two sets of psychoeducational measurements it is necessary to express the measures in comparable units. Figure 1 represents measured intelligence and measured achievement as normalized standard scores whose respective means are 100 and whose standard deviations are approximately 10.

It can be seen that of the four children who score a measured IQ between 115 and 125, two score educational quotients of the same order while two score EQs of between 105 and 115. The best prediction of the EQ of a child who scores an IQ of around 120 (i.e. between 115 and 125) is the average of the four EQ scores, i.e. about 115. This represents a regression of 5 points toward the common mean of 100.

For a child with a measured IQ of 100, it can be seen that the predicted EQs are symmetrically distributed around a mean of 100, so that an observed IQ of 100 predicts an EQ of 100. The EQs of the four students with measured IQs between 75 and 85, on the other hand, would regress toward the mean, as shown, their average expected EQ being around 85.

This regression of predicted score toward the mean is mathematically inevitable, the formula for predicting an individual's score on test Y from his/her observed score on test X being:

$$\bar{z}_y = r \cdot z_x$$

where $\bar{z}_y$ is predicted score on test Y;

z_x is observed score on test X, each being expressed as a standard score; and

r is the correlation between the tests X and Y .

In more conventional units, e.g. where the mean is 100 as in the WISC or Revised Stanford-Binet tests, and if distributions of both measured IQs and measured EQs have the same standard deviation, the regression equation becomes:

$$\overline{EQ} = r.(IQ - 100) + 100 \quad \dots\dots\dots (1)^*$$

where $\overline{EQ}$ is predicted EQ.

It is $\overline{EQ}$ that should be our reference point, not IQ. As Thorndike (1963) succinctly stated:

it is necessary to define "underachievement" as discrepancy of actual achievement from the predicted value, predicted on the basis of the regression equation between aptitude and achievement. A failure to recognize this regression effect has rendered questionable, if not meaningless, much of the research on "underachievement". (p. 13)

Fact #2: Unless scores on an Intelligence test and on an achievement test are perfectly correlated, it is inevitable that there will be positive and negative differences between pairs of test scores for individuals. Overall, the amount of observed "underachievement", defined in terms of test score difference, must be exactly balanced by the amount of "overachievement".

It is convenient and simple but, as will be presently demonstrated, quite invalid to accept a fixed discrepancy of (say) 15 points between measured IQ and measured EQ as an index of "educational underachievement". To point out the fallaciousness of such a simplistic definition is not new (e.g. Vernon, 1956;

* See pp. 20-22.

Thorndike, 1963; Reynolds, 1965; McLeod, 1968), but the dogma that intelligence is fixed, and pre-determined and measured by an intelligence test has dominated educational psychological thinking during the first half of the twentieth century (Hunt, 1962), and led naturally to the conclusion that intelligence tests provide an accurate measure of a child's fixed "capacity" which should be the target of individual educational achievement but which naturally cannot be exceeded.

The idea of a fixed discrepancy between IQ and EQ providing a valid index of significant underachievement by the individual is open to criticism on at least two psychometric counts:

1. In the first place, it can be seen from formula (1) above that if EQ is being predicted from (say) the WISC Performance Scale (which correlates approximately 0.5 with educational achievement), the predicted EQ is a child with a WISC Performance IQ of 140, is:

$$\overline{EQ} = 0.5(140 - 100) + 100, \text{ i.e. } 120.$$

Predicted EQ is, therefore, 20 points lower than observed IQ; more than a half of the children scoring a WISC Performance IQ of 140, therefore, would be labelled as "underachievers" if the criterion were adopted of defining an underachiever as a student whose educational quotient lagged more than 15 points behind measured WISC Performance IQ. Vernon (1956) pointed out more than twenty years ago that a child with an IQ of 115 has three times more chance than a child of IQ 85 of being selected for remedial reading classes if "reading retardation" is defined in terms of a fixed discrepancy between IQ and RQ. If $(IQ - EQ) > 10$ is taken as a definition of underachievement in the distribution illustrated in Figure 1, then the number of "underachievers" identified within each IQ band is:

IQ band:	<u>75 - 85</u>	<u>85 - 95</u>	<u>95 - 105</u>	<u>105 - 115</u>	<u>115 - 125</u>
No. of students:	4	16	24	16	4
No. of underachievers:	<u>0</u>	<u>2</u>	<u>4</u>	<u>4</u>	<u>2</u>
% of underachievers:	0	12.5	16.7	25	50

2. To understand the second error which is committed if a constant IQ/EQ discrepancy is accepted as an indicator of underachievement, it is necessary to consider the nature of standardized test scores. Ever since Wechsler pointed out the shortcomings of the old-fashioned quotients (e.g. IQ as "Mental Age" divided by chronological age) it has been usual practice to convert raw test scores into normalized standard scores. That is, a test is administered to a sample of children and, on the basis of their raw scores on the test, the children are placed in percentile rank order and the raw score associated with each percentile rank is converted into the corresponding standard score (z-score) of the normal curve of probability. In turn, these z-scores are translated into more familiar terms by assigning an arbitrary mean and standard deviation to the z-scores. For example, the WISC uses a mean of 100 and a standard deviation of 15, the Revised Stanford-Binet a mean of 100 and a standard deviation of 16, while the ITPA's mean and standard deviation are 36 and 6 respectively.

A simplified picture of the process by which IQs and EQs are derived as normalized standard scores is illustrated in Figure 11 below. ("IQ test" and "EQ test" refer to intelligence test and educational achievement tests respectively.) Figure 11 illustrates, using nine student scores, the process that would be employed using maybe a couple of thousand children in an actual test standardization.

As the two abilities being assessed are positively correlated, it is to be expected that students who do well on one test will tend to do well on the other, but no two tests (nor even the same test administered on two separate occasions)

Figure 11.

Conversion of percentile rank to standardized scores

Child	Rank order on:		Rank order expressed as percentile rank on:		Corresponding z-score* on:		Standardized score:** (Mean: 100 s.d.: 15)		(IQ-EQ)
	<u>IQ test</u>	<u>EQ test</u>	<u>IQ test</u>	<u>EQ test</u>	<u>IQ</u>	<u>EQ</u>	<u>IQ</u>	<u>EQ</u>	
A	1	1	90	90	1.28	1.28	119	119	0
B	2	5	80	50	0.84	0.00	113	100	+13
C		2	70	80	0.52	0.84	108	113	- 5
D	4	3	60	70	0.25	0.52	104	108	- 4
E	5	4	50	60	0.00	0.25	100	104	- 4
F	6	7	40	30	-0.25	-0.52	96	92	+ 4
G	7	9	30	10	-0.52	-1.28	92	81	+11
H	8	6	20	40	-0.84	-0.25	87	96	- 9
I	9	8	10	20	-1.28	-0.84	81	87	- 6
			Mean:		0.00	0.00	100	100	28-28 =0

* From normal probability curve

** Each student's raw score on IQ test and EQ test is translated into these "quotients".

will put students into precisely the same rank order. And, as there is a fixed number of students (and therefore a fixed number of ranks) in each sample, if one student slips down on one test relative to his or her rank on the other, then another student or students must inevitably move up in rank to maintain the overall balance.

Thus, nine children, ranked according to performance on an IQ test would probably display ranks on an educational achievement test similar to those shown in the third column of Figure 11. (The precise distribution of ranks is immaterial; the conclusions follow, irrespective of the particular rank distributions).

Columns 4 and 5 of Figure 11 show the rank orders converted to Percentile Ranks and the z-scores of columns 6 and 7 are derived from normal probability tables (e.g. Wiersma, 1975, p. 384). A mean of 100 and a standard deviation of 15 are applied to these z-scores to produce the Quotients of the next two columns.

The extreme right column of Figure 11 shows the difference between IQ and EQ and it can be seen that the 28 points of "underachievement" exhibited by B, F and G are exactly balanced by the "overachievement" of C, D, E, H and I.

This balance must always happen; that is, for those students whose EQ is lower than their IQ, there have to be students whose EQs are higher than their IQs because the overall population mean IQ and EQ is, by definition, 100.

Moreover, the number of "underachievers" and "overachievers" identified by a given discrepancy score will depend on the correlation between the predictor (IQ) test and the achievement (EQ) test. For example, Figures III and IV depict the relationships between Reading Age and Mental Age, when mental age is measured by verbal and non-verbal tests of intelligence respectively. It can be seen that if underachievement is defined as a deficiency of 15 points between IQ and RQ, then twice as many "underachievers" are identified if a non-verbal intelligence test is used as predictor, than is the case when a verbal test of

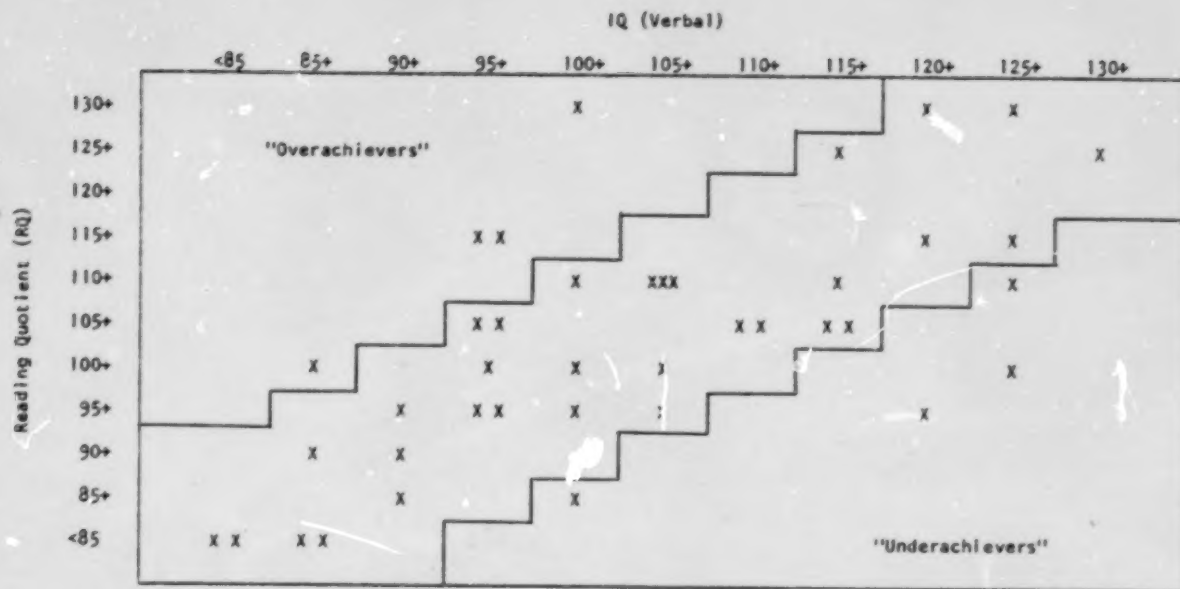
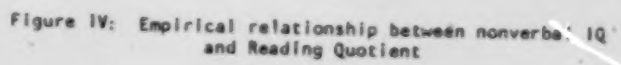


Figure III: Empirical relationship between verbal IQ and Reading Quotient



intelligence is used. The number of identified "overachievers" is, of course, correspondingly increased.

Fact #2: An empirical assessment of test performance is subject to an error of measurement which can be estimated.

Whenever an assessment is made by means of a standardized test (or by means of any other empirical means) the assessment is subject to error. This error can be estimated if the reliability and the distribution of scores on the test are known. The formula for the standard error of an individual test score is:

$$S.E. = S.D. \sqrt{1 - r_{11}}$$

where S.E. is the standard error of the individual test score;

S.D. is the standard deviation of scores on the test; and

r_{11} is the reliability of the test.

Thus, if a test, whose mean is 100 and standard deviation 15, has a reliability of 0.91, then the standard error of each individual test score is $15 \sqrt{1 - 0.91}$, i.e. 4.5.

From tables derived from the normal curve of probability, it may be ascertained that the chances are about 2 in 3, that a student's true test score is within one standard error of the observed score and the chances are about 1 in 20 that the true score will be within approximately two standard errors of the observed score.

Thus, it may be assumed that an observed individual score is correct to within:

± 4.5 points in 2 out of 3 cases; and

± 9 points in 19 out of 20 cases.

An error of this sort of order may be expected from an assessment of educational achievement and from a predictor (IQ) test. Therefore, if we are comparing a child's measured IQ with his EQ, the IQ is subject to an error of ± 9 points (if we want to

be sure "at the 5 per cent level of confidence" of his "true" IQ) and his EQ is also subject to an error of the order of 9 points. But, although "under-achievement" might be established by subtracting EQ from IQ, the errors of measurement may not obligingly be subtracted; in fact they accumulate. In the present illustration, the (IQ - EQ) difference would be subject to an error of 12 to 13 points, because the standard error of a difference between scores is given by the formula:

$$S.E. (IQ-EQ) = \sqrt{S.E. IQ^2 + S.E. EQ^2} \dots\dots\dots (2)$$

Hence, if a child has a measured IQ of 100 and a measured EQ of 90 on tests that have reliabilities of about 0.9 and the distribution of whose scores have standard deviations of 15 points, and if we want to be 95% certain of our facts, we have:

	<u>Observed</u>	<u>"True"</u>
IQ	100	91 to 109
EQ	90	81 to 99
IQ - EQ	10	-3 to +23

Examples in the literature of consequences of the myths and of the reality

1. Regression

For many years, there has been a series of arbitrary formulas designed to mitigate the gross errors which result from presenting mental age as the universal target and expectancy of educational achievement. In the euphoria of the early days of mental measurement, it was perhaps understandable for such giants as Burt (1937), and E.L. Thorndike and Gates (1929) to state that an achievement/capacity ratio of unity would serve as an index to evaluate student and teacher efficiency, but it is difficult to comprehend how in 1976, the Department of Education for the State of Michigan could regard mental age as the general criterion for educational expectancy -- for, shorn of its unnecessarily complicated format, that is precisely what the formula:

$$\text{Expected Grade} = \frac{CA \times IQ}{100} - 5$$

states. For, "expected achievement age" equals "expected achievement grade" plus 5, so

$$\text{Expected Achievement Age} = \frac{CA \times IQ}{100}$$

and, as -- according to the traditional formula, $IQ = \frac{MA}{CA} \times 100$, the Michigan formula reduces to:

$$\text{Expected Achievement Age} = MA.$$

There has generally been some attempt to amend expectancies generated by the naive formula used by Michigan to reduce the predicted achievement of students of high IQ, and to arrive at predictions which approximate more closely to the observed data which obey the law of regression. Bond and Clymer (see Bond and Tinker, 1967), for instance, produced the formula:

$$\overline{GE} = \left[\frac{\text{No. of years}}{\text{in school}} \right] \times \frac{IQ}{100} + 1.0$$

where $\overline{GE}$ is expected Grade Equivalent.

Thus, a child of IQ 120 who is aged 10.0 years, i.e. who is at the Grade 5.0 level, and who has completed four years of school, would have the following predicted achievements, according to the Michigan, and the Bond and Clymer formulae, respectively:

a) Michigan formula: $\overline{GE} = \frac{10 \times 120}{100} - 5$, i.e. 12 - 5, or Grade 7.0

b) Bond and Clymer: $\overline{GE} = 4 \times \frac{120}{100} + 1.0$, i.e. Grade 5.8.

A child of the same age, with a measured IQ of 80, would have the following predicted educational achievements, according to the same two formula:

a) Michigan: $\overline{GE} = \frac{10 \times 80}{100} - 5$, i.e. 8 - 5, or Grade 3.0

b) Bond and Clymer: $\overline{GE} = 4 \times \frac{80}{100} + 1.0$, i.e. Grade 4.2.

It can be seen that the formula of Bond and Clymer succeeds in producing a regression toward the mean for both able and less able students.

Harris (1970) produced a formula for predicting Reading Age from measured IQ. Instead of equating predicted reading age with measured Mental Age, Harris arbitrarily "diluted" mental age with a portion of Chronological Age, i.e.

$$\text{Expected Reading Age, or } \overline{RA} = \frac{2 \cdot MA + CA}{3}$$

The Harris formula has a similar effect as the Bond and Clymer formula in artificially producing a regression toward the mean. This may be illustrated with the same hypothetical children as in the previous example:

a) Child with IQ of 120: $\overline{RA} = \frac{2 \times 12 + 10}{3} = \frac{34}{3} = 11.3$,

i.e. Predicted Reading Grade = 6.3.

b) Child with IQ of 80: $\overline{RA} = \frac{2 \times 8 + 10}{3} = \frac{26}{3} = 8.7$,

i.e. Predicted Reading Grade = 3.7.

Again, there is regression from the unrealistic estimate of the Michigan "formula" toward the mean Grade Equivalent 5.0.

The Harris formula is of more than historical interest because, generalized from its original purpose of defining reading expectancy to the concept of learning disability, and subjected to further arbitrary amendment, it became the basis of the quantitative definition of learning disability proposed by the U.S. Department of Health, Education and Welfare in regulations related to Public Law 94-142. According to those regulations (Federal Register 1976) a child was to be defined as learning disabled if his/her grade equivalent in specified educational achievement was less than:

$$CA \times \left[\frac{IQ}{300} + 0.17 \right] - 2.5$$

This impressively mathematical-looking formula was derived simply, and arbitrarily, by dividing the arbitrary Harris prediction by two. In other words, a child was defined as learning disabled if his/her grade achievement level failed to reach a half of that predicted by the Harris reading formula.

According to Harris,

$$\overline{RA} = \frac{2xMA + CA}{3}$$

$$\text{or } \overline{GE} = \frac{2xMA + CA}{3} - 5$$

$$\text{So, a half of Grade Expectancy} = \frac{2xMA + CA}{6} - 2.5$$

$$= \frac{MA}{3} + \frac{CA}{6} - 2.5$$

$$\text{But } IQ = \frac{MA}{CA} \times 100, \text{ so } MA = \frac{CA \cdot IQ}{100}$$

$$\text{Hence, a half of Grade Expectancy} = \frac{CA \cdot IQ}{3 \cdot 100} + \frac{CA}{6} - 2.5$$

$$= CA \left[\frac{IQ}{300} + \frac{1}{6} \right] - 2.5$$

which is the Federal Register formula.

2. The unpalatable balance between underachievers and overachievers

Largely because of the influence of genetically-oriented psychologists such as Hall, Goddard, Terman and Gesell in the United States, and Burt in England, intelligence tests have been assumed to assess innate capacity (Hunt, 1962). According to such a view it is patently absurd to conceive that score -- on a test of educational achievement could possibly exceed score on a test of intelligence, or "capacity". The power of this dogma is demonstrated by the readiness with which many writers will accept test results at their face value when IQ exceeds EQ, but who cannot accept discrepancies between test scores when measured achievement exceeds IQ.

Otto, McMenemy and Smith (1973), for instance, set out the alleged scores of a Grade 7 class on a series of achievement and ability tests and make a number of individual initial diagnoses on the basis of the data. In the case of student #23 whose measured reading age is well below his mental age, the authors maintain that

"The -1.8 lag in RA appears clearly to identify this child as a corrective case in reading. Actually, he is close to being a disabled reader. Both further diagnosis and corrective or remedial reading are needed. (p. 109)

In the case of student #14 whose measured mental age is behind his measured achievement in reading and arithmetic, the authors are not so ready to accept the data. They recommend that

"it would be good to get a recheck on mental ability because the existing test score does not appear to be valid." (p. 108)

Perhaps the most remarkable evidence of the durability of the conviction that an individual's measured achievement cannot exceed his/her measured intelligenc

comes from Otto, McMenemy and Smith's treatment of students #17 and #25. Each of these students had an arithmetic age considerably higher than their measured mental age and reading age, which were quite compatible with each other.

"Both of these pupils have AAs that are more than two years above MA. The substantial gap strongly suggests that the mental ability score may be in error. While both pupils appear to be reading at about their expectancy level, reading achievement is substantially below arithmetic achievement. Further checking is needed. (p. 109)

Instead of seeing a couple of average students achieving at a good average grade level in reading, and with the welcome bonus of an aptitude for mathematics the authors contrive to suggest that, as arithmetic achievement is superior, then because so too true mental ability must be also, and so maybe we have a couple of retarded readers!

An approach to a solution

Errors of measurement, or virtue out of necessity

It has been common and respectable clinical practice (Schonell, 1942; Myklebust, 1967) to regard a discrepancy between measured intelligence and measured achievement as an identifying symptom of a learning disability. But to accept the discrepancy as an identifying symptom is not the same as asserting, or implying, that measured achievement level ought to be the same as that of measured intelligence (McLeod, 1968). Mathematical estimation of errors of measurement enables us to evaluate the significance of score differences and offers the possibility of a rational and defensible quantitative description of learning disability.

There are four identifiable steps in making a comparison between a student's measured IQ with his or her Educational Achievement Quotient (EQ):

- (1) The student's IQ is measured and the error of individual measured IQs can be estimated from the reliability of the IQ test.
- (2) The student's EQ is predicted from his/her IQ, because of the correlation between the intelligence test scores and Educational Achievement test scores.
 - (2a) If the standard deviation of the EQ distribution is known, a band of confidence for the individual predictions can be estimated.
- (3) The student's EQ is measured and the error of individual EQs can be estimated from the reliability of the educational achievement test.
- (4) The student's measured EQ is compared with the EQ predicted from the IQ test. The difference between measured and predicted EQs is subject to the compounded errors of predicted EQ and measured EQ.

Retracing the steps quantitatively:

- (1) Error of measured IQ:

For the present, it will be assumed that the predictor (IQ) test has perfect reliability, in order to keep the algebra reasonably easy to follow.

- (2) Prediction of EQ from IQ:

$$\bar{z}_E = r \cdot z_I \quad \dots\dots\dots (3)$$

where $\bar{z}_E$ is estimated educational achievement, expressed as a standard score;

r is the correlation between IQs and EQs; and

z_I is intelligence test score, expressed as a standard score

If intelligence and educational achievement are expressed as normalized standard scores, each with a mean of 100 and with standard deviations of σ_{IQ} and σ_{EQ} respectively, equation (3) becomes:

$$\frac{\bar{EQ} - 100}{\sigma_{EQ}} = r \cdot \frac{IQ - 100}{\sigma_{IQ}}$$

i.e. $\bar{EQ} = r \cdot \frac{\sigma_{EQ}}{\sigma_{IQ}} (IQ - 100) + 100 \quad \dots\dots\dots (4)$

where $\bar{EQ}$ is the individual's predicted educational achievement quotient.

- (2a) Standard error of
- $\bar{EQ}$
- :

The standard error of an individual EQ, estimated from IQ, is given by the formula:

$$(S.E.\bar{EQ}) = \sigma_{EQ} \sqrt{1-r^2} \quad \dots\dots\dots (5)$$

- (3) Assessment of the individual student's actual EQ:

$$\text{Standard error of the individual EQ, } (S.E.EQ) = \sigma_{EQ} \sqrt{1-r_{EQ}^2} \quad \dots\dots (6)$$

where σ_{EQ} = standard deviation of the EQ distribution; and

r_{EQ} = the reliability of the educational achievement test.

- (4) Comparison of actual and predicted EQ:

The difference between actual and observed EQ for an individual, from equation (4), is:

$$r \cdot \frac{\sigma_{EQ}}{\sigma_{IQ}} (IQ - 100) + 100 - EQ \quad \dots\dots\dots (7)$$

It is possible to evaluate whether this difference is significant, i.e. whether actual EQ falls significantly below prediction if the standard error of this difference is known. The difference is between two measures, the errors of each of which are independent, and therefore the overall error will be compounded according to equation (8).

$$SE_{\text{Difference (x-y)}}^2 = SE_x^2 + SE_y^2 \quad \dots\dots\dots (8)$$

The errors to be compounded are those of the individually assessed EQ (equation 6) and the predicted EQ (equation 5).

Thus, the standard error of the difference ($\overline{EQ} - EQ$) is given by the formula:

$$S.E._{\text{Diff.}}^2 = \sigma_{EQ}^2 (1-r_{EQ}^2) + \sigma_{EQ}^2 (1-r^2) \quad \dots\dots\dots (9)$$

Formulae (4) and (9) may be simplified considerably if scores on the intelligence test and educational achievement test have the same standard deviation. In this case, formula (4), which predicts an individual EQ from an observed IQ, becomes:

$$\overline{EQ} = r \cdot (IQ - 100) + 100 \quad \dots\dots\dots (1)$$

and formula (9), which estimates the standard error of the difference between predicted and actual EQ, becomes:

$$S.E._{\text{Diff.}}^2 = \sigma^2 (2 - r_{EQ} - r^2)$$

or

$$S.E._{\text{Diff.}} = \sigma \sqrt{2 - r_{EQ} - r^2} \quad \dots\dots\dots (9a)$$

In equations (1) and (9a),

- $\overline{EQ}$ is predicted Educational Achievement Quotient;
- r is the correlation between predictor and criterion, i.e. between intelligence test and educational achievement test;
- $S.E._{\text{Diff.}}$ is the standard error of the difference between observed and actual Educational Achievement Quotients;

σ is the standard deviation of scores on the educational achievement tests; and

r_{EQ} is the reliability of the educational achievement tests.

Equation (9a) is derived on the assumption that the reliability of the IQ test (r_{IQ}) is perfect, i.e. $r_{IQ} = 1$. In practice, of course, the IQ test reliability will not be perfect, in which case, formula (9a) becomes:

$$S.E. Diff. = \sigma \sqrt{2 - r_{EQ} - r_{IQ} \cdot r^2} \dots\dots\dots (9b)$$

The derivation of equation (9b) is contained in Appendix C.

If the distribution of test scores, reliabilities of the predictor and the criterion tests, and the intercorrelation between predictor test and criterion are known, it is possible by means of equations (1) and (9b) to estimate how frequently a measured EQ will be associated with the same individual's measured IQ. If, adopting a common statistical convention of regarding an observed event as "significant" if it would occur on fewer than one occasion in a hundred by chance, it is possible to define -- at least tentatively -- a learning disabled child as one whose EQ significantly lags behind the EQ predicted by his/her measured IQ. Table 1 depicts (a) EQs predicted from IQ according to Equation (1), assuming that both IQ and EQ distributions have means of 100 and standard deviations of 15, and that the correlation between IQ and EQ is +0.7, and (b) the actual EQs that would identify the lowest 0.5%, 1%, 2% and 5% respectively at each IQ level, assuming that the IQ test and the EQ test each has a reliability of 0.9.

Table 2 provides the same information as Table 1 except that Table 2 is based on an inter-test correlation of +0.5. Thus Table 2 may be regarded as approximating to a situation where educational achievement is predicted from a non-verbal test such as the WISC Performance Scale, while Table 1 would be more

IQ	Predicted EQ	Lower level of predicted Educational Achievement Quotient (EQ) to identify:			
		<u>5%</u>	<u>1%</u>	<u>2%</u>	<u>5%</u>
150	135	104	107	110	115
145	132	101	104	107	112
140	128	97	100	103	108
135	125	94	97	100	105
130	121	90	93	96	101
125	118	87	90	93	98
120	114	83	86	89	94
115	111	80	83	86	91
110	107	76	79	82	87
105	104	73	76	79	84
100	100	69	72	75	80
95	97	66	69	72	77
90	93	62	65	68	73
85	90	59	62	65	70
80	86	55	58	61	66
75	83	52	55	58	63

IQ test reliability = 0.9

EQ test reliability = 0.9

IQ/EQ correlation = 0.7

Table 1: EQ cutoffs for students of different measured
(verbal) IQs

IQ	Predicted EQ	Lower level of predicted Educational Achievement Quotient (EQ) to identify:			
		53	62	72	82
150	125	89	92	96	102
145	123	87	90	94	100
140	120	84	87	91	97
135	118	82	85	89	95
130	115	79	82	86	92
125	113	77	80	84	90
120	110	74	77	81	87
115	108	72	75	79	85
110	105	69	72	76	82
105	103	67	70	74	80
100	100	64	67	71	77
95	98	62	65	69	75
90	95	59	62	66	72
85	93	57	60	64	70
80	90	54	57	61	67
75	88	52	55	59	65

IQ test reliability = 0.9

EQ test reliability = 0.9

IQ/EQ correlation = 0.5

Table 2: EQ cutoffs for students of different measured
(non-verbal) IQs

applicable when prediction is made from a verbal test of intelligence, such as the WISC Verbal Scale or the Revised Stanford-Binet.

Although Tables 1 and 2 assume test reliabilities of 0.9, comparable tables can be constructed for any other reliabilities or test intercorrelations. (See Appendix A for BASIC Computer program.)

Quantitative description of learning disability when achievement scores are expressed in terms of Grade Equivalents:

Tables such as 1 and 2 which are expressed in terms of Quotients (normalized standard scores), are applicable at any age or grade level if the underlying assumptions are met that both IQs and EQs are expressed in comparable units, i.e. if the IQ and EQ distributions have the same mean and standard deviation.

If performances on two or more tests are to be made, the tests ought to use comparable units, otherwise we are likely to be trying to compare apples with pears. However, provided caution is exercised in interpretation, estimates can be derived of Achievement Grade levels which are significantly lower than those which would be predicted from a student's measured IQ.

In order to make these estimates, Equations (1) and (9b) must be modified so that EQs are replaced by Achievement Grade Equivalents. In order to do this, certain assumptions have to be made, viz.:

1. Achievement Grade Equivalent (GE) = EA - 5

2.
$$IQ = \frac{100 \times MA}{CA}$$

3.
$$EQ = \frac{100 \times EA}{CA}$$

Unfortunately, there is no guarantee, when such assumptions are made, that the distributions of measured intelligence and measured achievement will have the same variance, or that either of them will maintain the same variance at different

age levels. Indeed, it was precisely the fluctuation of IQ variance at different age levels, when IQ was calculated from MA/CA, that led Wechsler to replace this old-style quotient by a normalized standard score.

Tables 3 and 4 provide for a student of 9 years, 6 months, the same sort of information as do Tables 1 and 2, except that the cutoff achievement levels are expressed in terms of Grade Equivalents instead of EQs. More complete tables, for students between seven and fourteen years are contained in Appendix B. It must be emphasized that Tables 3 and 4, and the tables of Appendix B must be interpreted cautiously as approximations. Hopefully, they are reasonably close approximations -- but approximations nevertheless.

Suggested use of quantitative identification of learning disability

No child should ever be held to be eligible -- or ineligible -- for special educational treatment solely on the basis of some psychometric formula, whether it be the rigid imposition of a 70 IQ limit for educable mental retardation classes, the arbitrary but impressive-looking formula of the Federal Register regulations accompanying U.S. Public Law 94-142 to define learning disability or the more rational formulae developed in the present paper.

As has been pointed out earlier, the EQs or GEs listed in the various tables indicate the level of educational achievement below which it is unreasonable to conclude that the discrepancy between measured IQ and measured achievement is due to chance error of measurement or test unreliability. The "level of confidence" to be used by an educational jurisdiction will depend on (a) a deliberate professional decision and/or (b) the number of children that the administrative jurisdiction is prepared to recognize and provide with specialized help. The achievement levels in each of the columns of the Tables 1 through 4 identify the proportion of students indicated at the head of the particular column.

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	<u>.5%</u>	<u>1%</u>	<u>2%</u>	<u>5%</u>
150	4.8	5.1	5.4	5.9
145	4.5	4.8	5.1	5.6
140	4.2	4.5	4.8	5.3
135	3.8	4.1	4.5	4.9
130	3.5	3.8	4.1	4.6
125	3.2	3.5	3.8	4.3
120	2.9	3.1	3.5	3.9
115	2.5	2.8	3.1	3.6
110	2.2	2.5	2.8	3.3
105	1.9	2.1	2.5	2.9
100	1.5	1.8	2.1	2.6
95	1.2	1.5	1.8	2.3
90	.9	1.1	1.5	1.9
85	.5	.8	1.1	1.6
80	.2	.5	.8	1.3
75	0	.1	.5	.9

Correlation between intelligence and achievement
assumed to be 0.7;
IQ and EQ reliabilities assumed to be 0.9.

Table 3: Grade Equivalent cutoffs for children aged 9 years 6 months,
of different measured IQ.

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	<u>.52</u>	<u>.72</u>	<u>.82</u>	<u>.92</u>
150	3.4	3.8	4.1	4.7
145	3.2	3.5	3.9	4.4
140	3.0	3.3	3.7	4.2
135	2.7	3.1	3.4	4.0
130	2.5	2.8	3.2	3.7
125	2.3	2.6	2.9	3.5
120	2.0	2.3	2.7	3.3
115	1.8	2.1	2.5	3.0
110	1.5	1.9	2.2	2.8
105	1.3	1.6	2.0	2.5
100	1.1	1.4	1.8	2.3
95	.8	1.2	1.5	2.1
90	.6	.9	1.3	1.8
85	.4	.7	1.0	1.6
80	.1	.4	.8	1.4
75	0	.2	.6	1.1

Correlation between intelligence and achievement
assumed to be 0.5;

IQ and EQ reliabilities assumed to be 0.9.

Table 4: Grade Equivalent cutoffs for children aged 9 years 6 months,
of different measured IQ.

If, for example, a school jurisdiction is prepared to make provision for 2 per cent of its students as "learning disabled" then the EQs or GE's in the column headed "22" ought to identify 2 per cent of students whose actual achievement is significantly below prediction from IQ, in a school population whose mean IQ and EQ are 100 and where the assumptions on which the equations are based have been met. However, it would be educationally preferable to use a more generous criterion -- say, the achievement index in the column headed "52" -- as an initial screening device to help decide which children should be referred for more individualized diagnosis.

Any psychometric method of identifying learning disabled children, or potentially learning disabled children, should be based on tests which measure in comparable units, e.g. each test having a mean of 100 and a standard deviation of 15 for the population using the tests. The fact that an Intelligence test standardized in California and a reading test standardized in Britain might each have similar published means and standard deviations does not necessarily mean that these dimensions will be comparable in (say) Ontario, Canada. Educational jurisdictions would be well-advised to select acceptable aptitude and educational achievement tests, or develop their own, and to carry out local norming programs in order to establish the reliabilities, correlations and local measuring units, so that valid comparisons may be made.

Prediction of educational achievement should be made from both verbal (oral) and non-verbal aptitude tests according to tables custom-designed to fit empirical data from locally-normed tests. If prediction from the two types of aptitude tests yield conflicting results as to whether a student's educational achievement is unusually low for his/her measured intelligence, s/he should be given the benefit of further individual diagnosis.

Administrative designation of exceptionality

Although a purist clinician might express dismay, a judgment as to whether a child should be designated as learning disabled stems from what is essentially an administrative decision as to how many dollars are to be spent, and therefore how many children are to receive additional specialist assistance. Sometimes administrative decisions are made directly as to how many children will be recognized as learning disabled. Regulations issued in relation to the U.S. Public Law 94-142, for example, announced that up to 12 per cent of the school population would be recognized as educationally handicapped and, of these, up to one sixth, i.e. 2 per cent of the total population, would be recognized as learning disabled.

It might further be argued by an administrator that although a student's educational achievement level is unusually and significantly low for his/her measured intelligence, it is nevertheless within the range of achievement that the regular class teacher ought to be able to cope with unaided. For example, it might be contended that a teacher in an average Grade 4 room ought to be able to cope with reading achievement down to at least the Grade 2.5 level, and that special help is therefore not warranted for children with reading achievement above 2.5. Such a decision might be administratively logical and satisfying, but it is educationally and clinically questionable. For example, according to Table 4, if a nine-and-a-half year old child with non-verbal IQ of 135 was achieving at a Grade Equivalent level in reading of 2.7, the discrepancy between predicted and actual reading level would be so great that it might be expected in fewer than 0.5% of cases, i.e. less than once in 200 times. Such an atypical performance clearly warrants further investigation and diagnosis.

Conclusion

The development and judicious use of reliable and valid psychometric instruments can help toward the identification of children with specific learning problems and toward the efficient bringing to bear of skilled diagnostic and prescriptive services. The routine use of screening tests can help to bring to attention some children who might otherwise be overlooked. Kealy (1976) found, for example, that children from upper-socioeconomic homes had a significantly greater chance of having been brought to the attention of specialist educational diagnostic services than did children from lower socioeconomic families even though learning disability is to be found in all social strata. Presumably middle-class parents tend to be more articulate and demanding, and the aspirations and expectations on behalf of middle- and upper-class children are probably greater than that for lower-class children.

As Cruickshank has properly contended, identification and diagnosis of learning disability is an individual, human process. Yet there is a need for definitions of learning disability not only for the purpose of individual diagnosis, but for replicability of research and for administrative communication. Senf (1977) warns us that

We must be very careful to recognize the purpose of our definitions. Federal and state definitions serve primarily economic, administrative, and sociological functions. No definition of learning disabilities formulated at this level can be responsive to the diverse theoretical perspectives of either research or practice.

The purpose of the present paper is to suggest that, used responsibly, psychometricians can play a part in helping locate those children for whom more intensive individual diagnosis is indicated, and that the psychometrics involved can be rational and defensible.

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Appendix A

LD

BASIC Computer Program

```
100 PRINT "    *** Program to calculate the lower limit of estimated ***"  
110 PRINT "    educational achievement (expressed in terms of"  
120 PRINT "    Grade Equivalents) from individual IQs, according"  
130 PRINT "    to prescribed confidence criteria."  
140 PRINT  
150 PRINT  
160 PRINT "Predictor (i.e. IQ) and Educational Achievement tests"  
170 PRINT "have a population mean of ";  
180 INPUT M  
190 PRINT "And the population standard deviation is ";  
200 INPUT D  
210 PRINT "The reliability of the Intelligence test is ";  
220 INPUT R1  
230 PRINT "The reliability of the Educational Achievement test is ";  
240 INPUT R2  
250 PRINT "Correlation between IQ and educational achievement is"  
260 PRINT "estimated to be ";  
270 INPUT R  
280 PRINT "Tables to be printed at what intervals ? i.e. how many"  
290 PRINT "months apart ";  
300 INPUT S5  
310 PRINT "how many copies of printout ";  
320 INPUT C6  
330 FOR H8=1 TO C6  
340 DIM K[6],L[6],Z[6]  
350 MAT K=ZER  
360 MAT L=ZER  
370 Z[1]=2.575  
380 Z[2]=2.327  
390 Z[3]=2.054  
400 Z[4]=1.645  
410 DIM Q[6]  
420 MAT Q=ZER  
430 GOSUB 780  
440 GOSUB 780  
450 PRINT "IQ test reliability =";TAB(30);R1  
460 PRINT "EQ test reliability =";TAB(30);R2  
470 PRINT "IQ/EQ correlation =";TAB(30);R  
480 PRINT "Input '1' to start program"
```

```

490 INPUT K7
500 GOTO 530
510 GOSUB 780
520 GOSUB 960
530 FOR A=72 TO 168 STEP 55
540 Y=INT(A/12)
550 M1=A-Y*12
560 IF A/12<INT(A/12) THEN 570
570 PRINT TAB(20);Y;" years, ";M1;" months"
580 PRINT
590 PRINT " ", "Lower level of predicted achievement (in Grade"
600 PRINT "IQ", "Equivalent scores) in order to identify:"
610 PRINT " ", "0.5Z:          1Z:          2Z:          5Z:"
620 PRINT "----", "-----"
630 PRINT
640 FOR Q=150 TO 75 STEP -5
650 FOR K=1 TO 4
660 LCKJ=A*(R*(Q-M)+M-ZLKJ)*D*SQR(2-(R2+R1*R^2))/1200-5
670 IF LCKJ>.1 THEN 690
680 LCKJ=0
690 GOSUB 860
700 GOSUB 890
710 NEXT K
720 PRINT Q,L[1],L[2],L[3],L[4]
730 NEXT Q
740 GOSUB 780
750 NEXT A
760 NEXT HB
770 GOTO 1100
780 FOR I=1 TO 10
790 PRINT
800 NEXT I
810 RETURN
820 FOR J=1 TO 10
830 GOSUB 780
840 NEXT J
850 RETURN
860 LCKJ=INT(10*(LCKJ+.05))
870 LCKJ=LCKJ/10

```

```

880 RETURN
890 E=R*(Q-H)+H
900 E=INT(E+.5)
910 Q1=E-ZCK]*D*SQR(2-(R2+R1*R^2))
920 Q1=INT(Q1+.5)
930 QCK]=Q1
940 RETURN
950 PRINT
960 PRINT "The following table applies to children of any age."
970 PRINT
980 PRINT TAB(6); "Pred-" ; TAB(13); "Lower level of predicted Achievement"
990 PRINT " IQ" ; TAB(6); "icted" ; TAB(13); "Quotient to identify:"
1000 PRINT TAB(8); "EQ" ; TAB(13); ".5X" ; TAB(19); "1X" ; TAB(25); "2X" ; TAB(31); "3X"
1010 PRINT " ----" ; TAB(8); "----" ; TAB(13); "----" ; TAB(19); "----" ; TAB(25); "----" ; TAB(31); "----"
1020 PRINT
1030 FOR Q=150 TO 75 STEP -5
1040 FOR K=1 TO 4
1050 GOSUB 890
1060 NEXT K
1070 PRINT Q;E;Q[1];Q[2];Q[3];Q[4]
1080 NEXT Q
1090 RETURN
1100 END

```

Appendix B (1)

Grade Equivalent cutoffs for children aged 6 to 14 years

IQ/EQ correlation = 0.7

IQ reliability = 0.9

EQ reliability = 0.9

6 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	1.2	1.4	1.6	1.9
145	1	1.2	1.4	1.7
140	.8	1	1.2	1.5
135	.6	.8	1	1.3
130	.4	.6	.8	1.1
125	.2	.3	.5	.8
120	0	.1	.3	.6
115	0	0	.1	.4
110	0	0	0	.2
105	0	0	0	0
100	0	0	0	0
95	0	0	0	0
90	0	0	0	0
85	0	0	0	0
80	0	0	0	0
75	0	0	0	0

6 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	1.5	1.7	1.9	2.2
145	1.3	1.4	1.7	2
140	1	1.2	1.4	1.7
135	.8	1	1.2	1.5
130	.6	.8	1	1.3
125	.4	.6	.8	1.1
120	.2	.4	.6	.9
115	0	.1	.3	.7
110	0	0	.1	.4
105	0	0	0	.2
100	0	0	0	0
95	0	0	0	0
90	0	0	0	0
85	0	0	0	0
80	0	0	0	0
75	0	0	0	0

6 years, 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	1.7	1.9	2.1	2.5
145	1.5	1.7	1.9	2.2
140	1.3	1.5	1.7	2
135	1.1	1.3	1.5	1.8
130	.8	1	1.2	1.6
125	.6	.8	1	1.3
120	.4	.6	.8	1.1
115	.1	.3	.6	.9
110	0	.1	.3	.7
105	0	0	.1	.4
100	0	0	0	.2
95	0	0	0	0
90	0	0	0	0
85	0	0	0	0
80	0	0	0	0
75	0	0	0	0

6 years, 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	2	2.2	2.4	2.8
145	1.8	2	2.2	2.5
140	1.5	1.7	2	2.3
135	1.3	1.5	1.7	2.1
130	1.1	1.3	1.5	1.8
125	.8	1	1.2	1.6
120	.6	.8	1	1.3
115	.3	.5	.8	1.1
110	.1	.3	.5	.9
105	0	0	.3	.6
100	0	0	0	.4
95	0	0	0	.2
90	0	0	0	0
85	0	0	0	0
80	0	0	0	0
75	0	0	0	0

7 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5X:	1X:	2X:	3X:
---	-----	-----	-----	-----
150	2.3	2.5	2.7	3
145	2	2.2	2.5	2.8
140	1.8	2	2.2	2.6
135	1.5	1.7	2	2.3
130	1.3	1.5	1.7	2.1
125	1	1.2	1.5	1.8
120	.8	1	1.2	1.6
115	.5	.8	1	1.3
110	.3	.5	.7	1.1
105	0	.3	.5	.8
100	0	0	.2	.6
95	0	0	0	.4
90	0	0	0	.1
85	0	0	0	0
80	0	0	0	0
75	0	0	0	0

7 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5X:	1X:	2X:	3X:
---	-----	-----	-----	-----
150	2.5	2.7	3	3.3
145	2.3	2.5	2.7	3.1
140	2	2.2	2.5	2.8
135	1.8	2	2.2	2.6
130	1.5	1.7	2	2.3
125	1.2	1.5	1.7	2.1
120	1	1.2	1.5	1.8
115	.7	1	1.2	1.6
110	.5	.7	.9	1.3
105	.2	.4	.7	1.1
100	0	.2	.4	.8
95	0	0	.2	.5
90	0	0	0	.3
85	0	0	0	0
80	0	0	0	0
75	0	0	0	0

7 years, 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5X:	1X:	2X:	5X:
150	2.8	3	3.2	3.6
145	2.5	2.7	3	3.4
140	2.2	2.5	2.7	3.1
135	2	2.2	2.5	2.8
130	1.7	1.9	2.2	2.6
125	1.5	1.7	1.9	2.3
120	1.2	1.4	1.7	2
115	.9	1.2	1.4	1.8
110	.7	.9	1.1	1.5
105	.4	.6	.9	1.3
100	.1	.4	.6	1
95	0	.1	.4	.7
90	0	0	0	.5
85	0	0	0	.2
80	0	0	0	0
75	0	0	0	0

7 years, 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5X:	1X:	2X:	5X:
150	3	3.3	3.5	3.9
145	2.8	3	3.3	3.6
140	2.5	2.7	3	3.4
135	2.2	2.5	2.7	3.1
130	1.9	2.2	2.4	2.8
125	1.7	1.9	2.2	2.6
120	1.4	1.6	1.9	2.3
115	1.1	1.4	1.6	2
110	.9	1.1	1.4	1.7
105	.6	.8	1.1	1.5
100	.3	.6	.8	1.2
95	0	.3	.5	.9
90	0	0	.3	.7
85	0	0	0	.4
80	0	0	0	.1
75	0	0	0	0

8 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	3.3	3.5	3.8	4.2
145	3	3.3	3.5	3.9
140	2.7	3	3.2	3.6
135	2.5	2.7	3	3.4
130	2.2	2.4	2.7	3.1
125	1.9	2.1	2.4	2.8
120	1.6	1.9	2.1	2.5
115	1.3	1.6	1.8	2.2
110	1.1	1.4	1.6	2
105	.8	1.1	1.3	1.7
100	.5	.7	1	1.4
95	.2	.5	.7	1.1
90	0	.2	.4	.8
85	0	0	.2	.6
80	0	0	0	.3
75	0	0	0	0

8 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	3.6	3.8	4.1	4.5
145	3.3	3.5	3.8	4.2
140	3	3.2	3.5	3.9
135	2.7	2.9	3.2	3.6
130	2.4	2.6	2.9	3.3
125	2.1	2.4	2.6	3
120	1.8	2.1	2.3	2.8
115	1.5	1.8	2.1	2.5
110	1.2	1.5	1.8	2.2
105	1	1.2	1.5	1.9
100	.7	.9	1.2	1.6
95	.4	.6	.9	1.3
90	0	.3	.6	1
85	0	0	.3	.7
80	0	0	0	.4
75	0	0	0	.2

8 years: 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	3.8	4.1	4.3	4.8
145	3.5	3.8	4.1	4.5
140	3.2	3.5	3.8	4.2
135	2.9	3.2	3.5	3.9
130	2.6	2.9	3.2	3.6
125	2.3	2.6	2.9	3.3
120	2	2.3	2.6	3
115	1.7	2	2.3	2.7
110	1.4	1.7	2	2.4
105	1.1	1.4	1.7	2.1
100	.8	1.1	1.4	1.8
95	.5	.8	1.1	1.5
90	.2	.5	.8	1.2
85	0	.2	.5	.9
80	0	0	.2	.6
75	0	0	0	.3

8 years: 7 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	4.1	4.3	4.6	5.1
145	3.8	4	4.3	4.8
140	3.5	3.7	4	4.4
135	3.2	3.4	3.7	4.1
130	2.8	3.1	3.4	3.8
125	2.5	2.8	3.1	3.5
120	2.2	2.5	2.8	3.2
115	1.9	2.2	2.5	2.9
110	1.6	1.9	2.2	2.6
105	1.3	1.6	1.9	2.3
100	1	1.3	1.6	2
95	.7	1	1.3	1.7
90	.4	.7	.9	1.4
85	0	.4	.6	1.1
80	0	0	.3	.8
75	0	0	0	.5

9 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	4.3	4.6	4.9	5.3
145	4	4.3	4.6	5
140	3.7	4	4.3	4.7
135	3.4	3.7	4	4.4
130	3.1	3.3	3.6	4.1
125	2.8	3	3.3	3.8
120	2.4	2.7	3	3.5
115	2.1	2.4	2.7	3.1
110	1.8	2.1	2.4	2.8
105	1.5	1.8	2.1	2.5
100	1.2	1.4	1.7	2.2
95	.9	1.1	1.4	1.9
90	.5	.8	1.1	1.6
85	.2	.5	.8	1.3
80	0	.2	.5	.9
75	0	0	.2	.6

9 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	4.6	4.9	5.2	5.6
145	4.3	4.5	4.9	5.3
140	3.9	4.2	4.5	5
135	3.6	3.9	4.2	4.7
130	3.3	3.6	3.9	4.3
125	3	3.2	3.6	4
120	2.6	2.9	3.2	3.7
115	2.3	2.6	2.9	3.4
110	2	2.3	2.6	3
105	1.7	2	2.3	2.7
100	1.3	1.6	1.9	2.4
95	1	1.3	1.6	2.1
90	.7	1	1.3	1.7
85	.4	.7	1	1.4
80	0	.3	.6	1.1
75	0	0	.3	.8

9 years, 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	4.8	5.1	5.4	5.9
145	4.5	4.8	5.1	5.6
140	4.2	4.5	4.8	5.3
135	3.8	4.1	4.5	4.9
130	3.5	3.8	4.1	4.6
125	3.2	3.5	3.8	4.3
120	2.9	3.1	3.5	3.9
115	2.5	2.8	3.1	3.6
110	2.2	2.5	2.8	3.3
105	1.9	2.1	2.5	2.9
100	1.5	1.8	2.1	2.6
95	1.2	1.5	1.8	2.3
90	.9	1.1	1.5	1.9
85	.5	.8	1.1	1.6
80	.2	.5	.8	1.3
75	0	.1	.5	.9

9 years, 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	5.1	5.4	5.7	6.2
145	4.8	5.1	5.4	5.9
140	4.4	4.7	5	5.5
135	4.1	4.4	4.7	5.2
130	3.7	4	4.4	4.8
125	3.4	3.7	4	4.5
120	3.1	3.4	3.7	4.2
115	2.7	3	3.3	3.8
110	2.4	2.7	3	3.5
105	2	2.3	2.7	3.1
100	1.7	2	2.3	2.8
95	1.4	1.6	2	2.5
90	1	1.3	1.6	2.1
85	.7	1	1.3	1.8
80	.3	.6	.9	1.4
75	0	.3	.6	1.1

10 years, 0 months

IQ	Lower level of Predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	-----	-----	-----	-----
150	5.4	5.7	6	6.5
145	5	5.3	5.6	6.1
140	4.7	5	5.3	5.8
135	4.3	4.6	4.9	5.4
130	4	4.3	4.6	5.1
125	3.6	3.9	4.2	4.7
120	3.3	3.6	3.9	4.4
115	2.9	3.2	3.5	4
110	2.6	2.9	3.2	3.7
105	2.2	2.5	2.8	3.3
100	1.9	2.2	2.5	3
95	1.5	1.8	2.1	2.6
90	1.2	1.5	1.8	2.3
85	.8	1.1	1.4	1.9
80	.5	.8	1.1	1.6
75	.1	.4	.7	1.2

10 years, 3 months

IQ	Lower level of Predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	-----	-----	-----	-----
150	5.6	5.9	6.3	6.8
145	5.3	5.6	5.9	6.4
140	4.7	5.2	5.6	6.1
135	4.5	4.9	5.2	5.7
130	4.2	4.5	4.8	5.3
125	3.8	4.1	4.5	5
120	3.5	3.8	4.1	4.6
115	3.1	3.4	3.8	4.3
110	2.8	3.1	3.4	3.9
105	2.4	2.7	3	3.6
100	2	2.3	2.7	3.2
95	1.7	2	2.3	2.8
90	1.3	1.6	2	2.5
85	1	1.3	1.6	2.1
80	.6	.9	1.3	1.8
75	.2	.6	.9	1.4

10 years, 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	5.9	6.2	6.5	7.1
145	5.5	5.8	6.2	6.7
140	5.1	5.5	5.8	6.3
135	4.8	5.1	5.4	6
130	4.4	4.7	5.1	5.6
125	4	4.4	4.7	5.2
120	3.7	4	4.3	4.9
115	3.3	3.6	4	4.5
110	2.9	3.3	3.6	4.1
105	2.6	2.9	3.2	3.8
100	2.2	2.5	2.9	3.4
95	1.8	2.2	2.5	3
90	1.5	1.8	2.1	2.7
85	1.1	1.4	1.8	2.3
80	.7	1.1	1.4	1.9
75	.4	.7	1	1.6

10 years, 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	6.1	6.5	6.8	7.4
145	5.8	6.1	6.4	7
140	5.4	5.7	6.1	6.6
135	5	5.3	5.7	6.2
130	4.6	5	5.3	5.9
125	4.3	4.6	4.9	5.5
120	3.9	4.2	4.6	5.1
115	3.5	3.8	4.2	4.7
110	3.1	3.5	3.8	4.3
105	2.8	3.1	3.4	4
100	2.4	2.7	3.1	3.6
95	2	2.3	2.7	3.2
90	1.6	2	2.3	2.8
85	1.3	1.6	1.9	2.5
80	.9	1.2	1.6	2.1
75	.5	.8	1.2	1.7

11 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	6.4	6.7	7.1	7.6
145	6	6.3	6.7	7.3
140	5.6	6	6.3	6.9
135	5.2	5.6	5.9	6.5
130	4.9	5.2	5.6	6.1
125	4.5	4.8	5.2	5.7
120	4.1	4.4	4.8	5.3
115	3.7	4	4.4	5
110	3.3	3.7	4	4.6
105	2.9	3.3	3.6	4.2
100	2.6	2.9	3.2	3.8
95	2.2	2.5	2.9	3.4
90	1.8	2.1	2.5	3
85	1.4	1.7	2.1	2.6
80	1	1.3	1.7	2.3
75	.6	1	1.3	1.9

11 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	6.7	7	7.4	7.9
145	6.3	6.6	7	7.5
140	5.9	6.2	6.6	7.1
135	5.5	5.8	6.2	6.8
130	5.1	5.4	5.8	6.4
125	4.7	5	5.4	6
120	4.3	4.6	5	5.6
115	3.9	4.2	4.6	5.2
110	3.5	3.8	4.2	4.8
105	3.1	3.5	3.8	4.4
100	2.7	3.1	3.4	4
95	2.3	2.7	3	3.6
90	1.9	2.3	2.6	3.2
85	1.5	1.9	2.3	2.8
80	1.1	1.5	1.9	2.4
75	.8	1.1	1.5	2

11 years: 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	-----	-----	-----	-----
150	6.9	7.3	7.6	8.2
145	6.5	6.9	7.2	7.8
140	6.1	6.5	6.8	7.4
135	5.7	6.1	6.4	7
130	5.3	5.7	6	6.6
125	4.9	5.3	5.6	6.2
120	4.5	4.9	5.2	5.8
115	4.1	4.4	4.8	5.4
110	3.7	4	4.4	5
105	3.3	3.6	4	4.6
100	2.9	3.2	3.6	4.2
95	2.5	2.8	3.2	3.8
90	2.1	2.4	2.8	3.4
85	1.7	2	2.4	3
80	1.3	1.6	2	2.6
75	.9	1.2	1.6	2.2

11 years: 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	-----	-----	-----	-----
150	7.2	7.5	7.9	8.5
145	6.8	7.1	7.5	8.1
140	6.4	6.7	7.1	7.7
135	5.9	6.3	6.7	7.3
130	5.5	5.9	6.3	6.9
125	5.1	5.5	5.9	6.5
120	4.7	5.1	5.5	6
115	4.3	4.7	5	5.6
110	3.9	4.2	4.6	5.2
105	3.5	3.8	4.2	4.8
100	3.1	3.4	3.8	4.4
95	2.7	3	3.4	4
90	2.2	2.6	3	3.6
85	1.8	2.2	2.6	3.2
80	1.4	1.8	2.2	2.8
75	1	1.4	1.8	2.3

12 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	----	----	----	----
150	7.4	7.8	8.2	8.8
145	7	7.4	7.8	8.4
140	6.6	7	7.4	8
135	6.2	6.5	6.9	7.5
130	5.8	6.1	6.5	7.1
125	5.3	5.7	6.1	6.7
120	4.9	5.3	5.7	6.3
115	4.5	4.9	5.3	5.9
110	4.1	4.4	4.8	5.4
105	3.7	4	4.4	5
100	3.2	3.6	4	4.6
95	2.8	3.2	3.6	4.2
90	2.4	2.8	3.2	3.8
85	2	2.3	2.7	3.3
80	1.6	1.9	2.3	2.9
75	1.1	1.5	1.9	2.5

12 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	----	----	----	----
150	7.7	8.1	8.5	9.1
145	7.3	7.6	8	8.7
140	6.8	7.2	7.6	8.2
135	6.4	6.8	7.2	7.8
130	6	6.4	6.8	7.4
125	5.6	5.9	6.3	6.9
120	5.1	5.5	5.9	6.5
115	4.7	5.1	5.5	6.1
110	4.3	4.6	5	5.7
105	3.8	4.2	4.6	5.2
100	3.4	3.8	4.2	4.8
95	3	3.4	3.8	4.4
90	2.6	2.9	3.3	3.9
85	2.1	2.5	2.9	3.5
80	1.7	2.1	2.5	3.1
75	1.3	1.6	2	2.7

12 years, 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	8	8.3	8.7	9.4
145	7.5	7.9	8.3	8.9
140	7.1	7.5	7.9	8.5
135	6.6	7	7.4	8.1
130	6.2	6.6	7	7.6
125	5.8	6.1	6.6	7.2
120	5.3	5.7	6.1	6.7
115	4.9	5.3	5.7	6.3
110	4.5	4.8	5.2	5.9
105	4	4.4	4.8	5.4
100	3.6	4	4.4	5
95	3.1	3.5	3.9	4.6
90	2.7	3.1	3.5	4.1
85	2.3	2.6	3.1	3.7
80	1.8	2.2	2.6	3.2
75	1.4	1.8	2.2	2.8

12 years, 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	8.2	8.6	9	9.7
145	7.8	8.2	8.6	9.2
140	7.3	7.7	8.1	8.8
135	6.9	7.3	7.7	8.3
130	6.4	6.8	7.2	7.9
125	6	6.4	6.8	7.4
120	5.5	5.9	6.3	7
115	5.1	5.5	5.9	6.5
110	4.6	5	5.5	6.1
105	4.2	4.6	5	5.6
100	3.8	4.1	4.6	5.2
95	3.3	3.7	4.1	4.7
90	2.9	3.2	3.7	4.3
85	2.4	2.8	3.2	3.9
80	2	2.4	2.8	3.4
75	1.5	1.9	2.3	3

13 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	----	----	----	----
150	8.5	8.9	9.3	9.9
145	8	8.4	8.8	9.5
140	7.6	8	8.4	9
135	7.1	7.5	7.9	8.6
130	6.7	7	7.5	8.1
125	6.2	6.6	7	7.7
120	5.7	6.1	6.6	7.2
115	5.3	5.7	6.1	6.8
110	4.8	5.2	5.7	6.3
105	4.4	4.8	5.2	5.9
100	3.9	4.3	4.7	5.4
95	3.5	3.9	4.3	4.9
90	3	3.4	3.8	4.5
85	2.6	3	3.4	4
80	2.1	2.5	2.9	3.6
75	1.6	2	2.5	3.1

13 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	----	----	----	----
150	8.7	9.1	9.6	10.2
145	8.3	8.7	9.1	9.8
140	7.8	8.2	8.6	9.3
135	7.3	7.7	8.2	8.8
130	6.9	7.3	7.7	8.4
125	6.4	6.8	7.3	7.9
120	6	6.4	6.8	7.5
115	5.5	5.9	6.3	7
110	5	5.4	5.9	6.5
105	4.6	5	5.4	6.1
100	4.1	4.5	4.9	5.6
95	3.6	4	4.5	5.1
90	3.2	3.6	4	4.7
85	2.7	3.1	3.5	4.2
80	2.2	2.6	3.1	3.7
75	1.8	2.2	2.6	3.3

13 years: 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	-----	-----	-----	-----
150	9	9.4	9.8	10.5
145	8.5	8.9	9.4	10
140	8	8.5	8.9	9.6
135	7.6	8	8.4	9.1
130	7.1	7.5	8	8.6
125	6.6	7	7.5	8.2
120	6.2	6.6	7	7.7
115	5.7	6.1	6.5	7.2
110	5.2	5.6	6.1	6.7
105	4.7	5.1	5.6	6.3
100	4.3	4.7	5.1	5.8
95	3.8	4.2	4.7	5.3
90	3.3	3.7	4.2	4.9
85	2.8	3.3	3.7	4.4
80	2.4	2.8	3.2	3.9
75	1.9	2.3	2.8	3.4

13 years: 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	-----	-----	-----	-----
150	9.3	9.7	10.1	10.8
145	8.8	9.2	9.6	10.3
140	8.3	8.7	9.2	9.8
135	7.8	8.2	8.7	9.4
130	7.3	7.7	8.2	8.9
125	6.8	7.3	7.7	8.4
120	6.4	6.8	7.2	7.9
115	5.9	6.3	6.8	7.4
110	5.4	5.8	6.3	7
105	4.9	5.3	5.8	6.5
100	4.4	4.9	5.3	6
95	4	4.4	4.8	5.5
90	3.5	3.9	4.3	5
85	3	3.4	3.9	4.6
80	2.5	2.9	3.4	4.1
75	2	2.4	2.9	3.6

14 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	-----	-----	-----	-----
150	9.5	9.9	10.4	11.1
145	9	9.4	9.9	10.6
140	8.5	9	9.4	10.1
135	8	8.5	8.9	9.6
130	7.6	8	8.4	9.1
125	7.1	7.5	7.9	8.6
120	6.6	7	7.5	8.2
115	6.1	6.5	7	7.7
110	5.6	6	6.5	7.2
105	5.1	5.5	6	6.7
100	4.6	5	5.5	6.2
95	4.1	4.5	5	5.7
90	3.6	4.1	4.5	5.2
85	3.1	3.6	4	4.7
80	2.7	3.1	3.5	4.2
75	2.2	2.6	3	3.7

Appendix B (II)

Grade Equivalent cutoffs for children aged 6 to 14 years

IQ/EQ correlation = 0.5

IQ reliability = 0.9

EQ reliability = 0.9

6 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	.3	.5	.6	1.1
145	.2	.4	.6	1
140	0	.2	.5	.8
135	0	0	.3	.7
130	0	0	.2	.5
125	0	0	0	.4
120	0	0	0	.2
115	0	0	0	0
110	0	0	0	0
105	0	0	0	0
100	0	0	0	0
95	0	0	0	0
90	0	0	0	0
85	0	0	0	0
80	0	0	0	0
75	0	0	0	0

6 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	.6	.8	1	1.4
145	.4	.6	.9	1.2
140	.2	.5	.7	1.1
135	0	.3	.5	.9
130	0	.1	.4	.7
125	0	0	.2	.6
120	0	0	0	.4
115	0	0	0	.3
110	0	0	0	.1
105	0	0	0	0
100	0	0	0	0
95	0	0	0	0
90	0	0	0	0
85	0	0	0	0
80	0	0	0	0
75	0	0	0	0

6 years, 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5X:	1X:	2X:	5X:
----	-----	-----	-----	-----
150	.8	1	1.3	1.6
145	.6	.8	1.1	1.5
140	.5	.7	.9	1.3
135	.3	.5	.8	1.1
130	.1	.4	.6	1
125	0	.2	.4	.8
120	0	0	.3	.6
115	0	0	.1	.5
110	0	0	0	.3
105	0	0	0	.2
100	0	0	0	0
95	0	0	0	0
90	0	0	0	0
85	0	0	0	0
80	0	0	0	0
75	0	0	0	0

6 years, 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5X:	1X:	2X:	5X:
----	-----	-----	-----	-----
150	1	1.2	1.5	1.9
145	.8	1.1	1.3	1.7
140	.7	.9	1.2	1.5
135	.5	.7	1	1.4
130	.3	.6	.8	1.2
125	.2	.4	.6	1
120	0	.2	.5	.9
115	0	0	.3	.7
110	0	0	.1	.5
105	0	0	0	.4
100	0	0	0	.2
95	0	0	0	0
90	0	0	0	0
85	0	0	0	0
80	0	0	0	0
75	0	0	0	0

7 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	1.2	1.5	1.7	2.1
145	1	1.3	1.6	2
140	.9	1.1	1.4	1.8
135	.7	.9	1.2	1.6
130	.5	.8	1	1.4
125	.3	.6	.9	1.3
120	.2	.4	.7	1.1
115	0	.2	.5	.9
110	0	0	.3	.7
105	0	0	.2	.6
100	0	0	0	.4
95	0	0	0	.2
90	0	0	0	0
85	0	0	0	0
80	0	0	0	0
75	0	0	0	0

7 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	1.4	1.7	2	2.4
145	1.3	1.5	1.8	2.2
140	1.1	1.3	1.6	2
135	.9	1.2	1.4	1.8
130	.7	1	1.2	1.7
125	.5	.8	1.1	1.5
120	.4	.6	.9	1.3
115	.2	.4	.7	1.1
110	0	.2	.5	.9
105	0	0	.3	.8
100	0	0	.2	.6
95	0	0	0	.4
90	0	0	0	.2
85	0	0	0	0
80	0	0	0	0
75	0	0	0	0

7 years, 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	1.7	1.9	2.2	2.6
145	1.5	1.7	2	2.5
140	1.3	1.6	1.8	2.3
135	1.1	1.4	1.7	2.1
130	.9	1.2	1.5	1.9
125	.7	1	1.3	1.7
120	.5	.8	1.1	1.5
115	.4	.6	.9	1.3
110	.2	.4	.7	1.1
105	0	.2	.5	1
100	0	0	.3	.8
95	0	0	.2	.6
90	0	0	0	.4
85	0	0	0	.2
80	0	0	0	0
75	0	0	0	0

7 years, 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	1.9	2.2	2.5	2.9
145	1.7	2	2.3	2.7
140	1.5	1.8	2.1	2.5
135	1.3	1.6	1.9	2.3
130	1.1	1.4	1.7	2.1
125	.9	1.2	1.5	1.9
120	.7	1	1.3	1.7
115	.5	.8	1.1	1.5
110	.3	.6	.9	1.3
105	.1	.4	.7	1.2
100	0	.2	.5	1
95	0	0	.3	.8
90	0	0	.1	.6
85	0	0	0	.4
80	0	0	0	.2
75	0	0	0	0

8 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	-----	-----	-----	-----
150	2.1	2.4	2.7	3.2
145	1.9	2.2	2.5	3
140	1.7	2	2.3	2.8
135	1.5	1.8	2.1	2.6
130	1.3	1.6	1.9	2.4
125	1.1	1.4	1.7	2.2
120	.9	1.2	1.5	2
115	.7	1	1.3	1.8
110	.5	.8	1.1	1.6
105	.3	.6	.9	1.4
100	.1	.4	.7	1.2
95	0	.2	.5	1
90	0	0	.3	.8
85	0	0	0	.6
80	0	0	0	.4
75	0	0	0	.2

8 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	-----	-----	-----	-----
150	2.3	2.6	2.9	3.4
145	2.1	2.4	2.7	3.2
140	1.9	2.2	2.5	3
135	1.7	2	2.3	2.8
130	1.5	1.8	2.1	2.6
125	1.3	1.6	1.9	2.4
120	1.1	1.4	1.7	2.2
115	.9	1.2	1.5	2
110	.7	1	1.3	1.8
105	.5	.8	1.1	1.6
100	.3	.6	.9	1.3
95	0	.4	.7	1.1
90	0	.1	.5	.9
85	0	0	.3	.7
80	0	0	0	.5
75	0	0	0	.3

8 years, 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	2.6	2.8	3.2	3.7
145	2.3	2.6	3	3.5
140	2.1	2.4	2.8	3.2
135	1.9	2.2	2.5	3
130	1.7	2	2.3	2.8
125	1.5	1.8	2.1	2.6
120	1.3	1.6	1.9	2.4
115	1.1	1.4	1.7	2.2
110	.9	1.1	1.5	2
105	.6	.9	1.3	1.8
100	.4	.7	1.1	1.5
95	.2	.5	.8	1.3
90	0	.3	.6	1.1
85	0	0	.4	.9
80	0	0	.2	.7
75	0	0	0	.5

8 years, 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	2.8	3.1	3.4	3.9
145	2.6	2.9	3.2	3.7
140	2.3	2.6	3	3.5
135	2.1	2.4	2.8	3.3
130	1.9	2.2	2.5	3
125	1.7	2	2.3	2.8
120	1.5	1.8	2.1	2.6
115	1.2	1.5	1.9	2.4
110	1	1.3	1.7	2.2
105	.8	1.1	1.4	1.9
100	.6	.9	1.2	1.7
95	.4	.7	1	1.5
90	.2	.5	.8	1.3
85	0	.2	.6	1.1
80	0	0	.4	.9
75	0	0	.1	.6

9 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	3	3.3	3.7	4.2
145	2.8	3.1	3.4	3.9
140	2.5	2.9	3.2	3.7
135	2.3	2.6	3	3.5
130	2.1	2.4	2.8	3.3
125	1.9	2.2	2.5	3
120	1.6	2	2.3	2.8
115	1.4	1.7	2.1	2.6
110	1.2	1.5	1.9	2.4
105	1	1.3	1.6	2.1
100	.7	1.1	1.4	1.9
95	.5	.8	1.2	1.7
90	.3	.6	1	1.5
85	0	.4	.7	1.2
80	0	.2	.5	1
75	0	0	.3	.8

9 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	3.2	3.5	3.9	4.4
145	3	3.3	3.7	4.2
140	2.8	3.1	3.4	4
135	2.5	2.8	3.2	3.7
130	2.3	2.6	3	3.5
125	2.1	2.4	2.7	3.3
120	1.8	2.2	2.5	3
115	1.6	1.9	2.3	2.8
110	1.4	1.7	2	2.6
105	1.1	1.5	1.8	2.3
100	.9	1.2	1.6	2.1
95	.7	1	1.4	1.9
90	.4	.8	1.1	1.7
85	.2	.5	.9	1.4
80	0	.3	.7	1.2
75	0	0	.4	1

9 years, 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	---	---	---	---
150	3.4	3.8	4.1	4.7
145	3.2	3.5	3.9	4.4
140	3	3.3	3.7	4.2
135	2.7	3.1	3.4	4
130	2.5	2.8	3.2	3.7
125	2.3	2.6	2.9	3.5
120	2	2.3	2.7	3.3
115	1.8	2.1	2.5	3
110	1.5	1.9	2.2	2.8
105	1.3	1.6	2	2.5
100	1.1	1.4	1.8	2.3
95	.8	1.2	1.5	2.1
90	.6	.9	1.3	1.8
85	.4	.7	1	1.6
80	.1	.4	.8	1.4
75	0	.2	.6	1.1

9 years, 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	---	---	---	---
150	3.7	4	4.4	4.9
145	3.4	3.8	4.1	4.7
140	3.2	3.5	3.9	4.4
135	2.9	3.3	3.6	4.2
130	2.7	3	3.4	4
125	2.4	2.8	3.2	3.7
120	2.2	2.5	2.9	3.5
115	2	2.3	2.7	3.2
110	1.7	2.1	2.4	3
105	1.5	1.8	2.2	2.7
100	1.2	1.6	1.9	2.5
95	1	1.3	1.7	2.3
90	.7	1.1	1.5	2
85	.5	.8	1.2	1.8
80	.3	.6	1	1.5
75	0	.3	.7	1.3

10 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	3.9	4.2	4.6	5.2
145	3.6	4	4.4	4.9
140	3.4	3.7	4.1	4.7
135	3.1	3.5	3.9	4.4
130	2.9	3.2	3.6	4.2
125	2.6	3	3.4	3.9
120	2.4	2.7	3.1	3.7
115	2.1	2.5	2.9	3.4
110	1.9	2.2	2.6	3.2
105	1.6	2	2.4	2.9
100	1.4	1.7	2.1	2.7
95	1.1	1.5	1.9	2.4
90	.9	1.2	1.6	2.2
85	.6	1	1.4	1.9
80	.4	.7	1.1	1.7
75	.1	.5	.9	1.4

10 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	4.1	4.5	4.9	5.4
145	3.9	4.2	4.6	5.2
140	3.6	4	4.3	4.9
135	3.3	3.7	4.1	4.7
130	3.1	3.4	3.8	4.4
125	2.8	3.2	3.6	4.2
120	2.6	2.9	3.3	3.9
115	2.3	2.7	3.1	3.7
110	2.1	2.4	2.8	3.4
105	1.8	2.2	2.6	3.1
100	1.5	1.9	2.3	2.9
95	1.3	1.6	2	2.6
90	1	1.4	1.8	2.4
85	.8	1.1	1.5	2.1
80	.5	.9	1.3	1.9
75	.3	.6	1	1.6

10 years: 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5X:	1X:	2X:	5X:
150	4.3	4.7	5.1	5.7
145	4.1	4.4	4.8	5.4
140	3.8	4.2	4.6	5.2
135	3.5	3.9	4.3	4.9
130	3.3	3.6	4	4.7
125	3	3.4	3.8	4.4
120	2.8	3.1	3.5	4.1
115	2.5	2.9	3.3	3.9
110	2.2	2.6	3	3.6
105	2	2.3	2.7	3.3
100	1.7	2.1	2.5	3.1
95	1.4	1.8	2.2	2.8
90	1.2	1.5	1.9	2.6
85	.9	1.3	1.7	2.3
80	.7	1	1.4	2
75	.4	.8	1.2	1.8

10 years: 7 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5X:	1X:	2X:	5X:
150	4.6	4.9	5.3	6
145	4.3	4.7	5.1	5.7
140	4	4.4	4.8	5.4
135	3.7	4.1	4.5	5.2
130	3.5	3.9	4.3	4.9
125	3.2	3.6	4	4.6
120	2.9	3.3	3.7	4.3
115	2.7	3	3.5	4.1
110	2.4	2.8	3.2	3.8
105	2.1	2.5	2.9	3.5
100	1.9	2.2	2.7	3.3
95	1.6	2	2.4	3
90	1.3	1.7	2.1	2.7
85	1.1	1.4	1.8	2.5
80	.8	1.2	1.6	2.2
75	.5	.9	1.3	1.9

11 years: 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	4.8	5.2	5.6	6.2
145	4.5	4.9	5.3	5.9
140	4.2	4.6	5	5.7
135	4	4.3	4.8	5.4
130	3.7	4.1	4.5	5.1
125	3.4	3.8	4.2	4.8
120	3.1	3.5	3.9	4.6
115	2.9	3.2	3.7	4.3
110	2.6	3	3.4	4
105	2.3	2.7	3.1	3.7
100	2	2.4	2.8	3.5
95	1.8	2.1	2.6	3.2
90	1.5	1.9	2.3	2.9
85	1.2	1.6	2	2.6
80	.9	1.3	1.7	2.4
75	.7	1	1.5	2.1

11 years: 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	5	5.4	5.8	6.5
145	4.7	5.1	5.5	6.2
140	4.4	4.8	5.3	5.9
135	4.2	4.5	5	5.6
130	3.9	4.3	4.7	5.3
125	3.6	4	4.4	5.1
120	3.3	3.7	4.1	4.8
115	3	3.4	3.9	4.5
110	2.7	3.1	3.6	4.2
105	2.5	2.9	3.3	3.9
100	2.2	2.6	3	3.7
95	1.9	2.3	2.7	3.4
90	1.6	2	2.4	3.1
85	1.3	1.7	2.2	2.8
80	1.1	1.5	1.9	2.5
75	.8	1.2	1.6	2.2

11 years, 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	5.2	5.6	6.1	6.7
145	4.9	5.3	5.8	6.4
140	4.6	5	5.5	6.1
135	4.4	4.8	5.2	5.9
130	4.1	4.5	4.9	5.6
125	3.8	4.2	4.6	5.3
120	3.5	3.9	4.3	5
115	3.2	3.6	4	4.7
110	2.9	3.3	3.8	4.4
105	2.6	3	3.5	4.1
100	2.3	2.7	3.2	3.8
95	2.1	2.5	2.9	3.6
90	1.8	2.2	2.6	3.3
85	1.5	1.9	2.3	3
80	1.2	1.6	2	2.7
75	.9	1.3	1.7	2.4

11 years, 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	5.4	5.9	6.3	7
145	5.1	5.6	6	6.7
140	4.9	5.3	5.7	6.4
135	4.6	5	5.4	6.1
130	4.3	4.7	5.1	5.8
125	4	4.4	4.8	5.5
120	3.7	4.1	4.5	5.2
115	3.4	3.8	4.2	4.9
110	3.1	3.5	4	4.6
105	2.8	3.2	3.7	4.3
100	2.5	2.9	3.4	4
95	2.2	2.6	3.1	3.7
90	1.9	2.3	2.8	3.5
85	1.6	2	2.5	3.2
80	1.3	1.7	2.2	2.9
75	1	1.4	1.9	2.6

12 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	5.7	6.1	6.5	7.2
145	5.4	5.8	6.2	6.9
140	5.1	5.5	5.9	6.6
135	4.8	5.2	5.6	6.3
130	4.5	4.9	5.3	6
125	4.2	4.6	5	5.7
120	3.9	4.3	4.7	5.4
115	3.6	4	4.4	5.1
110	3.3	3.7	4.1	4.8
105	3	3.4	3.8	4.5
100	2.7	3.1	3.5	4.2
95	2.4	2.8	3.2	3.9
90	2.1	2.5	2.9	3.6
85	1.8	2.2	2.6	3.3
80	1.5	1.9	2.3	3
75	1.2	1.6	2	2.7

12 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	5.9	6.3	6.8	7.5
145	5.6	6	6.5	7.2
140	5.3	5.7	6.2	6.9
135	5	5.4	5.9	6.6
130	4.7	5.1	5.6	6.3
125	4.4	4.8	5.3	6
120	4	4.5	4.9	5.6
115	3.7	4.2	4.6	5.3
110	3.4	3.9	4.3	5
105	3.1	3.6	4	4.7
100	2.8	3.3	3.7	4.4
95	2.5	2.9	3.4	4.1
90	2.2	2.6	3.1	3.8
85	1.9	2.3	2.8	3.5
80	1.6	2	2.5	3.2
75	1.3	1.7	2.2	2.9

12 years, 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5X:	1X:	2X:	5X:
150	6.1	6.5	7	7.7
145	5.8	6.2	6.7	7.4
140	5.5	5.9	6.4	7.1
135	5.2	5.6	6.1	6.8
130	4.9	5.3	5.8	6.5
125	4.5	5	5.5	6.2
120	4.2	4.7	5.1	5.9
115	3.9	4.4	4.8	5.6
110	3.6	4	4.5	5.2
105	3.3	3.7	4.2	4.9
100	3	3.4	3.9	4.6
95	2.7	3.1	3.6	4.3
90	2.4	2.8	3.3	4
85	2	2.5	3	3.7
80	1.7	2.2	2.6	3.4
75	1.4	1.9	2.3	3.1

12 years, 7 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5X:	1X:	2X:	5X:
150	6.3	6.8	7.3	8
145	6	6.5	6.9	7.7
140	5.7	6.1	6.6	7.4
135	5.4	5.8	6.3	7
130	5.1	5.5	6	6.7
125	4.7	5.2	5.7	6.4
120	4.4	4.9	5.4	6.1
115	4.1	4.5	5	5.8
110	3.8	4.2	4.7	5.4
105	3.5	3.9	4.4	5.1
100	3.1	3.6	4.1	4.8
95	2.8	3.3	3.8	4.5
90	2.5	2.9	3.4	4.2
85	2.2	2.6	3.1	3.9
80	1.9	2.3	2.8	3.5
75	1.5	2	2.5	3.2

13 years, 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	6.6	7	7.5	8.2
145	6.2	6.7	7.2	7.9
140	5.9	6.4	6.9	7.6
135	5.6	6	6.5	7.3
130	5.3	5.7	6.2	6.9
125	4.9	5.4	5.9	6.6
120	4.6	5.1	5.6	6.3
115	4.3	4.7	5.2	6
110	4	4.4	4.9	5.6
105	3.6	4.1	4.6	5.3
100	3.3	3.8	4.3	5
95	3	3.4	3.9	4.7
90	2.7	3.1	3.6	4.3
85	2.3	2.8	3.3	4
80	2	2.5	3	3.7
75	1.7	2.1	2.6	3.4

13 years, 3 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
150	6.8	7.2	7.7	8.5
145	6.4	6.9	7.4	8.2
140	6.1	6.6	7.1	7.8
135	5.8	6.2	6.8	7.5
130	5.5	5.9	6.4	7.2
125	5.1	5.6	6.1	6.8
120	4.8	5.2	5.8	6.5
115	4.5	4.9	5.4	6.2
110	4.1	4.6	5.1	5.9
105	3.8	4.3	4.8	5.5
100	3.5	3.9	4.4	5.2
95	3.1	3.6	4.1	4.9
90	2.8	3.3	3.8	4.5
85	2.5	2.9	3.4	4.2
80	2.1	2.6	3.1	3.9
75	1.8	2.3	2.8	3.5

13 years, 6 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5X:	1X:	2X:	5X:
150	7	7.5	8	8.8
145	6.7	7.1	7.6	8.4
140	6.3	6.8	7.3	8.1
135	6	6.5	7	7.7
130	5.6	6.1	6.6	7.4
125	5.3	5.8	6.3	7.1
120	5	5.4	6	6.7
115	4.6	5.1	5.6	6.4
110	4.3	4.8	5.3	6.1
105	4	4.4	4.9	5.7
100	3.6	4.1	4.6	5.4
95	3.3	3.8	4.3	5
90	2.9	3.4	3.9	4.7
85	2.6	3.1	3.6	4.4
80	2.3	2.7	3.3	4
75	1.9	2.4	2.9	3.7

13 years, 9 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5X:	1X:	2X:	5X:
150	7.2	7.7	8.2	9
145	6.9	7.4	7.9	8.7
140	6.5	7	7.5	8.3
135	6.2	6.7	7.2	8
130	5.8	6.3	6.8	7.6
125	5.5	6	6.5	7.3
120	5.2	5.6	6.2	7
115	4.8	5.3	5.8	6.6
110	4.5	4.9	5.5	6.3
105	4.1	4.6	5.1	5.9
100	3.8	4.3	4.8	5.6
95	3.4	3.9	4.4	5.2
90	3.1	3.6	4.1	4.9
85	2.8	3.2	3.6	4.5
80	2.4	2.9	3.4	4.2
75	2.1	2.5	3.1	3.9

14 years; 0 months

IQ	Lower level of predicted achievement (in Grade Equivalent scores) in order to identify:			
	0.5%:	1%:	2%:	5%:
---	-----	-----	-----	-----
150	7.4	7.9	8.5	9.3
145	7.1	7.6	8.1	8.9
140	6.7	7.2	7.8	8.6
135	6.4	6.9	7.4	8.2
130	6	6.5	7.1	7.9
125	5.7	6.2	6.7	7.5
120	5.3	5.8	6.4	7.2
115	5	5.5	6	6.8
110	4.6	5.1	5.7	6.5
105	4.3	4.8	5.3	6.1
100	3.9	4.4	5	5.8
95	3.6	4.1	4.6	5.4
90	3.2	3.7	4.3	5.1
85	2.9	3.4	3.9	4.7
80	2.5	3	3.6	4.4
75	2.2	2.7	3.2	4

Derivation of formula (9b), page 23

The author acknowledges his indebtedness to Thorndike (1963, pp. 70-71), for the following derivation.

In the following equations, all scores are expressed as derivations from the mean, i.e.

$$\begin{aligned} x_{IQ} &= IQ - 100, \quad \text{where IQ is measured IQ,} \\ x_{EQ} &= EQ - 100, \quad \text{where EQ is measured EQ,} \\ \bar{x}_{EQ} &= \bar{EQ} - 100, \quad \text{where } \bar{EQ} \text{ is predicted EQ.} \\ \bar{x}_{EQ} &= r \cdot \frac{\sigma_{EQ}}{\sigma_{IQ}} \cdot x_{IQ} \quad \dots\dots\dots (C.1) \end{aligned}$$

$$\text{Therefore, } x_D = (x_{EQ} - \bar{x}_{EQ}) = x_{EQ} - r \cdot \frac{\sigma_{EQ}}{\sigma_{IQ}} \cdot x_{IQ} \quad \dots\dots\dots (C.2)$$

where x_D = the discrepancy between predicted and measured achievement,

and where r = correlation between predictor and criterion tests.

It will be assumed that scores on the predictor test (IQ) and on the criterion test (EQ) have the same standard deviation, σ . Equation (C.2) therefore becomes:

$$x_D = x_{EQ} - r \cdot x_{IQ} \quad \dots\dots\dots (C.2a)$$

The standard deviation of discrepancy scores is:

$$\sigma_D = \sigma \sqrt{1 - r^2} \quad \dots\dots\dots (C.3)$$

The correlation between two independent measures of the discrepancy between predicted and measured achievement, $r_{DD'}$, is given by:

$$r_{DD'} = \frac{1}{N} \cdot \frac{\sum x_D \cdot x_{D'}}{\sigma_D \cdot \sigma_D}$$

$$= \frac{1}{N} \cdot \frac{\Sigma(x_{EQ} - rx_{IQ})(x_{EQ'} - rx_{IQ'})}{(\sigma \sqrt{1-r^2})(\sigma \sqrt{1-r^2})}$$

$$= \frac{1}{N} \frac{(\Sigma x_{EQ} \cdot x_{EQ'} - r \Sigma x_{IQ} x_{EQ'} - r \Sigma x_{IQ'} x_{EQ} + r^2 \Sigma x_{IQ} \cdot x_{IQ'})}{\sigma^2(1-r^2)} \dots (C.4)$$

But $r_{EQ \cdot EQ'} = \frac{\Sigma x_{EQ} \cdot x_{EQ'}}{N \cdot \sigma_{EQ} \cdot \sigma_{EQ'}}$

Therefore,

$$\frac{1}{N} \Sigma x_{EQ} \cdot x_{EQ'} = \sigma_{EQ} \cdot \sigma_{EQ'} \cdot r_{EQ \cdot EQ'}$$

i.e. $\frac{1}{N} \Sigma x_{EQ} \cdot x_{EQ'} = \sigma^2 \cdot r_{EQ \cdot EQ'} \dots \dots \dots (C.5)$

Similarly,

$$\frac{1}{N} \Sigma x_{IQ} \cdot x_{EQ} = \sigma^2 \cdot r \dots \dots \dots (C.6)$$

and

$$\frac{1}{N} \Sigma x_{IQ} \cdot x_{IQ'} = \sigma^2 \cdot r_{IQ \cdot IQ'} \dots \dots \dots (C.7)$$

Substituting (C.5), (C.6) and (C.7) in (C.4),

$$r_{DD'} = \frac{\sigma^2 \cdot r_{EQ \cdot EQ'} - \sigma^2 r^2 - \sigma^2 r^2 + \sigma^2 \cdot r^2 \cdot r_{IQ \cdot IQ'}}{\sigma^2(1-r^2)}$$

$$= \frac{r_{EQ \cdot EQ'} - 2r^2 + r^2 \cdot r_{IQ \cdot IQ'}}{1-r^2} \dots \dots \dots (C.8)$$

Since $r_{DD'}$ is the correlation between two independent measures of the discrepancy between measured and predicted achievement, equation (C.8) represents an estimate of the reliability of the discrepancy measure. Similarly, $r_{EQ \cdot EQ'}$ and

$r_{IQ \cdot IQ}$ are coefficients of reliability of the EQ test and IQ test respectively.

Hence r_D , r_{EQ} , r_{IQ} will be used to denote the respective reliability co-efficients of discrepancy, EQ and IQ scores; equation (C.8) becomes:

$$r_D = \frac{r_{EQ} - 2r^2 + r^2 \cdot r_{IQ}}{1-r^2} \quad \text{..... (C.8)}$$

The standard error of measurement of an individual score, i.e. variability which would occur in repeated measures of the same specimen due solely to inherent error of measurement, is given by the general formula:

$$S.E. \text{ measurement} = S_x \sqrt{1 - r_x} \quad \text{..... (C.9)}$$

where S_x is the standard deviation of the variable, and

r_x is the reliability co-efficient of the variable.

By substituting (C.3) and (C.8a) in (C.9), the standard error of measures of discrepancy between predictor and criterion, is given by the equation:

$$S.E. \text{ meas.} = (\sigma \sqrt{1-r^2}) \left(\sqrt{1 - \frac{r_{EQ} - 2r^2 + r^2 \cdot r_{IQ}}{1-r^2}} \right) \quad \text{..... (C.10)}$$

$$\begin{aligned} &= \sigma \sqrt{\frac{(1-r^2)(1-r^2 - r_{EQ} + 2r^2 - r^2 \cdot r_{IQ})}{(1-r^2)}} \\ &= \sigma \sqrt{1 + r^2 - r_{EQ} - r^2 \cdot r_{IQ}} \quad \text{..... (C.11)} \end{aligned}$$

Equation (C.11) represents the standard error of variability in discrepancy scores that arise purely from errors of measurement.

It remains to allow for errors that arise in estimating one score from another.

The standard error of an estimate of EQ from measured IQ, is:

$$SE_{est.} = \sigma \sqrt{1 - r^2} \quad \dots\dots\dots (C.12)$$

where σ and r are as previously defined.

The errors of estimate and errors of measurement are independent, and therefore a discrepancy score between measured EQ and an EQ estimated from a measured IQ, will be subject to a total error of S.E. _{total}, where:

$$S.E._{total}^2 = S.E._{meas.}^2 + S.E._{est.}^2 \quad \dots\dots\dots (C.13)$$

Substituting (C.11) and (C.12) in (C.13),

$$\begin{aligned} S.E._{total}^2 &= \sigma^2(1 + r^2 - r_{EQ}^2 - r^2 \cdot r_{IQ} + 1 - r^2) \\ &= \sigma^2(2 - r_{EQ}^2 - r^2 \cdot r_{IQ}) \end{aligned}$$

$$\text{i.e.} \quad S.E._{total} = \sigma \sqrt{2 - r_{EQ}^2 - r^2 \cdot r_{IQ}} \quad \dots\dots\dots (C.14)$$

Equation (C.14) is presented in the body of the present monograph as equation (9b) and is the equation on which the various tables are based.